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# An Analysis of Copyright Protection Strategy With Customers Category and Network Externality

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## Abstract

Illegal reproduction is increasingly becoming a major concern of companies and the society. Previous research has shown when network effect is strong, piracy could be beneficial for firms. However, some researchers got that strong network effects can sometimes lead to a firm choosing higher levels of copyright protection. How to choose the investment strategy for firms in this society with prevalent piracy? There are two strategies: no copyright protection and setting copyright protection. We address two questions in a monopoly and duopoly setting. Frist, what effects the attractiveness of each of the two strategies? Second, under which conditions will any particular strategy dominate another? We show that in a monopoly setting, firms prefer not to take a copyright protection with higher level of network effect and more support-piracy consumers. In a duopoly setting, the equilibrium of game theory is at the choice of the strategy of copyright protection.

**Key words:** Piracy; Network externality; Copyright protection strategy; Customer category

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## INTRODUCTION

Recently, more and more illegal reproduction of products is becoming a large concern for both the society and the industry. In 2010, the Business Software Alliance said that 42% of software is pirated, which leads to an estimated loss of more than $59 billion to firms. However, copying of intellectual property is not limited to software. Piracy of music and movies is a major concern to the society, and some researchers have suggested that it will lead to radical changes in the industry (Moul, 2006). With growth of Internet, piracy is becoming even more prevalent because copying of intellectual properties is becoming easier and more difficult to prevent. And some companies have also tried to combat piracy by making their products more difficult to cope using digital rights management software. For example, Intuit incorporated a feature in its 2003 TurboTax such that the software could only be installed on one computer and Windows XP has an activation code; and some other companies never take any measure to prevent the piracy.

If piracy can increase the market size, when exist network externality, then company may get some income because of piracy. Recently some researchers have done some work about this. For example Conner and Rumelt (1991) and Takeyama (1994) assume that copying by individuals confers a network externality that benefit users of the product. Shy and Thisse (1999) extent the monopoly results of Conner and Rumelt (1991) and Takeyama (1994) to a duopoly framework and divided the consumers into two categories, high-valuation consumers who do not copy and low-valuation consumers who potentially can copy. Their results show that firms can benefit by not protecting their software if net- work externality is strong with allowing piracy increases the market size. When piracy can increase the market size, piracy is beneficial for the firms. But if piracy couldn't increase market size, in a saturated market or market with limited growth opportunity, some previous researchers have examined this situation. Jain (2008) examined if the market is saturated and there is limited opportunity for market growth, higher network effect can actually lead to higher

levels of copyright protection. Some other researchers showed that a corporation may choose different copyright strategy in different place with different piracy rate, for example, Ye (2006) have examined Microsoft has different strategy in different country and region. He will choose high levels of copyright protection if there has high piracy rate, vice versa. So we can see that different ratio of consumers would affect the choice of firms. And the purpose of this paper is examining the effect of ratio of consumers to the strategy of firms.

This paper base on Shy and Thisse (1999), reexamine the effect of network and initial ratio of consumers to the choice of firms' copyright protection strategy, when the market is saturated and there is limited opportunity for market growth. But they do not examined the choice of copyright protection strategy for firms with the different ratio of customers. We divide the consumers into two categories, no-support-piracy consumers who think the quality of pirated product is low and pirate is immorality and support-piracy consumers who think the pirate can replace the legal copy (see jain 2008 and Shy and Thisse 1999). When company chooses copyright protect, pirate could not satisfy the support- piracy consumers, so some of these consumers may support the legal copy. And these consumers become the no-support-piracy consumers. This paper examines two simple situations: Oligopoly and Duopoly. First, we use the Hotelling model to examine the situation of Oligopoly, then use the game model for Duopoly. We contrast two strategies: (a) no copyright protection. (b) Setting copy- right protection. We conclude that: firms prefer to not take a copyright protection with higher level of network effect and more support-piracy consumers under the first situation. And firms will have more opportunity to take the copyright protection with high level of network effect. Under the second situation, whenever what copyright protection one firm choose, the other one will choose a level of copyright protection.

## 1. MODEL AND ANALYSIS

### 1.1 Oligopoly

We first consider a monopolist selling to differentiated consumers. And there exist so- me other pirated corporation. We assume that the monopolist (firm 1) is at one end of the Hotelling line, in particular, at 0.

We assume that there are two segments of consumers. The first segment of consumers do not support pirated product. The second of consumers support pirated product. We don't take the cost of product into account. We use Hotelling model, and quote the conventional maximization to compare benefit. We also assume that a firm couldn't pursue benefit maximization after entering one market but to grab market share (Ye, 2006). So our purpose is comparing the benefit under pursue market share maximization.

Let $\alpha$ denotes level of copyright protection where $\alpha \in (0,1)$. $\beta$ denotes initial ratio of no-support-pirate consumers where $\beta \in (0,1)$, the other is $1-\beta$. we assume that firms take $\alpha$ level of copyright protection to defense pirate, then there is $(1-\beta)\alpha$ second segment consumers turn to support the legal copy because of lacking pirated product. We assume that all market demand have been completely satisfy (Sun, Xie, & Cao, 2006). Most intellectual product has network effect, and its fixed cost is high ,its marginal cost is low. So we assume its marginal cost is 0.

This paper contrast two strategies: (a) No copyright protection. Under this strategy, whenever there exist pirate or not, company never choose the copyright protection. (b) Setting copyright protection. Under this strategy, company must choose some levels of copyright protection to defense pirate. Each consumer demands either 1 or 0 unit of the product. And an individual will buy the product if the resulting surplus is nonnegative. And the market is saturated and there is limited opportunity of market growth.

#### 1.1.1 Decision Behavior of Strategy I

We assume that there exist a city line and the length is 1. The consumers are homogeneous along the line. And let $x$ denote the location of consumer. The distance between $x$ and endpoint is the cost of transport (jain 2006). The reservation price of a consumer, $x$, is defined $U=K+\gamma Q-x$ (Jain, 2006; Sun, Xie, & Cao, 2004), where $K$ ($0 \le K \le 1$) is the quality of product. $\gamma$ ($0<\gamma<1$) measures the strength of the net- work effect. $Q$ is the expected network size, and $Q=1$.

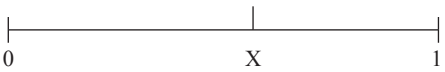


**Figure 1**
**City Line**

Let $P_{ij}$、$\pi_{ij}$ denote the price and income, where let i denotes the industry $i \in \{1,2\}$, j denotes the strategy, $j \in \{1,2\}$. The monopolist must demand the first segment consumers, so the price of product must satisfy the following condition:

$$P_{11} \le K_1 + \gamma Q - \beta$$

So the income of the monopolist is $\pi_{11} = P_{11} * \beta$.

We assume the quality is 1, $K_1 = 1$. So we have:

***Lemma 1.*** If the monopolist take strategy I, then its best decision is: when $P_{11}^* = K_1 + \gamma Q - \beta$, the monopolist will get the best income:

$$\pi_{11}^* = \beta(1+\gamma-\beta) \quad (1)$$

#### 1.1.2 Decision Behavior of Strategy II

With piracy, the monopolist will take a copyright protection strategy and take $\alpha$ level of copyright protection. Then there a $(1-\beta)g\alpha$ of the second segment consumers turn to support the legal copy. The monopolist

must pay for this, we assume this cost is $m\alpha^2, m \in (0,1)$ (Jain, 2006). The monopolist must demand all the first segment consumers and the consumers no more support- piracy of the second segment consumers, so the price of product must satisfy the following condition: $P_{12} \le K_1 + \gamma Q - (\beta + (1-\beta)\alpha)$. And its income function is:

$$\pi_{12} = P_{12}(\beta + (1-\beta)\alpha) - m\alpha^2$$

Then we have:

***Lemma 2.*** If the monopolist take strategy II, then its best decision is: when $P_{12}^* = K_1 + \gamma Q - (\beta + (1-\beta)\alpha)$.

The monopolist will get the best income:

$$\pi_{12}^* = (1 + \gamma - (\beta + (1-\beta)\alpha))(\beta + (1-\beta)\alpha) - m\alpha^2 \quad (2)$$

Now we compare the profits under these two strategy:

$$\pi_{11}^* - \pi_{12}^* = ((1-\beta)^2 + m)\alpha^2 + (2\beta - 1 - \gamma)(1-\beta)\alpha \quad (3)$$

***Proposition 1.*** **(1)** If $2\beta - 1 - \gamma \ge 0$, or $\beta \ge \frac{1+\gamma}{2}$, the firm will take strategy I. (2) If $2\beta - 1 - \gamma < 0$, or $\beta < \frac{1+\gamma}{2}$, (i) if $0 < \gamma \le m$,the firm will take strategy II and take a low levels of copyright protection.(ii) if $m < \gamma < 1$, ① when $0 < \beta < \beta_1$, the firm will take strategy II; ② when $\beta_1 \le \beta \le \frac{1+\gamma}{2}$, the firm will take strategy II and take a low levels of copyright protection. Where $\beta_1 = \frac{1 + \gamma - \sqrt{(1+\gamma)^2 + 4(m-\gamma)}}{2}$.

The ratio of first segment of consumers is beyond one half, without network effect, the firm adopt strategy II. When there exist network effect, as the network effect increasing, even the ratio of first segment of consumers is beyond one half, the monopolist may choose copyright protection strategy. That is to say the monopolist have larger probability to take copyright protection strategy with net- work effect. If the strength of network effect is strong and the first segment is low, then whenever copyright protection the monopolist takes, the benefit of strategy II always be more than strategy I's. This is because strong network effect will cause high reservation price of consumers, then the price of product could be high, and if the first segment consumers is beyond one half, the monopolist will take copyright protection to increase the output of its product for more income.

***Character 1.*** When network effect become stronger, the monopolist prefer to choose strategy II.

In a monopoly, when $\beta < \frac{1+\gamma}{2}$, the monopolist will take copyright protection. We know that if $\gamma \to 1$ (network effect become strong), the monopolist has more opportunity to take copyright protection strategy to win more benefit because of $\beta$ has increased. So the monopolist has bigger possibility to choose strategy II with much stronger network effect. That is because much stronger net- work effect result in higher production price, which will refuse some consumers to buy its product. And in order to get benefit maximization the monopolist must take copyright protection to strive for more consumers.

## 1.2 Duopoly

We now assume that there is a second firm in the market at the other end of the Hotelling line, i.e., at 1. And we assume the two firms are homogeneous, just have the same product and share the market. The other assumption are very similar to those of the monopoly case. Let $P_{ijk}$ and $\pi_{ijk}$ denote the price and benefit of firms, where the sub-script i is used to denoted the number of firms in the market, j denotes the strategy, k is the firm. And we assume the firms choose the copyright protection strategy in sequential.

### 1.2.1 All Choose Strategy I

Under this situation, we know that the firms have $\beta$ consumers, so each have $\frac{\beta}{2}$. Follow 1.1.1, we have:

**Lemma 3.** If all firms choose strategy I, then their best decisions are: when firm 1 and firm 2 have the price:

$$P_{211}^* = K_1 + \gamma Q - \frac{\beta}{2}$$

and $P_{212}^* = K_2 + \gamma Q - \frac{\beta}{2}$

The two firms will get the best income:

$$\pi_{211}^* = \frac{\beta}{2}(K_1 + \gamma Q - \frac{\beta}{2}) \quad (4)$$

$$\pi_{212}^* = \frac{\beta}{2}(K_2 + \gamma Q - \frac{\beta}{2}) \quad (5)$$

### 1.2.2 Firm 1 Chooses Strategy II, Firm 2 Chooses Strategy I

Firm 1 chooses strategy II, then firm 1 can get $\frac{\beta}{2} + \alpha(1-\beta)$ of market share, where $\alpha(1-\beta)$ is the consumers who turn to support the legal copy from the second segment. And the market share of firm 2 is —. Follow 1.1.1 and 1.1.2, we know:

***Lemma 4.*** If Firm 1 chooses Strategy II, firm 2 chooses Strategy I, then their best decisions are: when firm 1 and firm 2 have the price: $P_{221}^* = K_1 + \gamma Q - (\frac{\beta}{2} + \alpha(1-\beta))$ and $P_{212}^* = K_2 + \gamma Q - \frac{\beta}{2}$, the two firms will get the best income:

$$\pi_{221}^* = (K_1 + \gamma Q - (\frac{\beta}{2} + \alpha(1-\beta)))(\frac{\beta}{2} + \alpha(1-\beta)) \quad (6)$$

$$\pi_{212}^{*}=\frac{\beta}{2}(K_2+\gamma Q-\frac{\beta}{2}) \quad (7)$$

However, because two firms are homogeneous, the situation of Firm 1 chooses Strategy I, firm 2 chooses Strategy II is the same as this situation. So we no longer discuss this situation.

**1.2.3 All Choose Strategy II**

When firm 2 take $\alpha'$ level of copyright protection, firm 2 will have $(1-\beta)\alpha'$ proportion of second segment consumers. And firm 1 gets $(1-\alpha')(1-\beta)\alpha''$ proportion of second segment consumers. Because of sequential choice, we use $P'_{221}$ denotes the optimal price of firm 1. Then we have:

***Lemma 5.*** If all firms choose strategy II, then their best decisions are: when firm 2 has the price:

$P_{222}^{*}=K_2+\gamma Q-(\frac{\beta}{2}+\alpha'(1-\beta))$, then firm 2 gets the optimal income:

$$\pi_{222}=(K_2+\gamma Q-(—+\alpha'(1-\beta)))$$

$$\bullet(\frac{\beta}{2}+\alpha'(1-\beta))-m\alpha'^{2} \quad (8)$$

When firm 1 has the optimal price: $P_{221}'^{*}=K_1+\gamma Q-(\frac{\beta}{2}+\alpha''(1-\alpha')(1-\beta))$, firm 1 gets the optimal income, use $\pi'_{221}$ to denote it:

$$\pi_{221}'^{*}=(K_1+\gamma Q-(\frac{\beta}{2}+\alpha''(1-\alpha')(1-\beta)))$$

$$\bullet(\frac{\beta}{2}+\alpha''(1-\alpha')(1-\beta))-m\alpha''^{2} \quad (9)$$

This is a game model. The partners is the two firms. We assume $K_1=K_2=1$, because we think the two firms are homogeneous.

(1) When firm 2 chooses strategy I, and firm 1 chooses strategy I, the optimal income of firm 1 is $\pi_{211}^{*}=\frac{\beta}{2}(1+\gamma-\frac{\beta}{2})$. If firm 1 chooses strategy II, its optimal income is $\pi_{221}^{*}=(1+\gamma-(\frac{\beta}{2}+\alpha(1-\beta)))(\frac{\beta}{2}+\alpha(1-\beta))-m\alpha^{2}$.

Now we compare the two optimal incomes, then we have:

$$\pi_{221}^{*}-\pi_{211}^{*}=-((1-\beta)^2+m)\alpha^2+(1+\gamma-\beta)(1-\beta)\alpha \quad (10)$$

***Proposition 2.*** In Duopoly, when firm 2 chooses strategy I, (1) if $\gamma>m$, ①when $\beta>1-\frac{m}{\gamma}$, firm 1 will take strategy II, and he will take a low level of copyright protection. ② When $\beta<1-\frac{m}{\gamma}$, firm 1 will take strategy II. (2) If $\gamma\le m$, when $\beta\in(0,1)$, firm 1 will choose a low level of copyright protection.

So we know when one of duopoly chooses strategy I-Without any measure, then another also chooses strategy II- Setting copy- right protection. Along coefficient *m* increasing, the probability of firm choosing copy-right protection is becoming low. That is because that coefficient *m* is cost coefficient, if m is becoming high, firms will decrease the level of copyright protection to reduce the cost.

(2) When firm 2 chooses strategy II, and firm 1 chooses strategy I, then the optimal income of firm 1 is $\pi_{211}^{*}=\frac{\beta}{2}(1+\gamma-\frac{\beta}{2})$. firm1 chooses strategy II, its optimal income is $\pi_{221}'^{*}=(1+\gamma-(\frac{\beta}{2}+\alpha''(1-\alpha')(1-\beta)))$ $\bullet(\frac{\beta}{2}+\alpha''(1-\alpha')(1-\beta))-m\alpha''^{2}$.

Now we compare the two optimal incomes, then we have:

$$\pi_{221}'^{*}-\pi_{211}^{*}=-(m+(1-\beta)^2(1-\alpha')^2)\alpha''^{2}+$$

$$(1+\gamma-\beta)(1-\beta)(1-\alpha')\alpha'' \quad (11)$$

***Proposition 3.*** In Duopoly, (1) if $\gamma>m$, and when firm 2 chooses a low level of copyright protection, ① if $\beta<\beta_2$, firm 1 also take strategy II. ② If $\beta\ge\beta_2$, firm 1 will choose a low level of copyright protection. When firm 2 chooses a high level of copyright protection, firm 1 will take a low level of copyright protection. Where

$$\beta_2=\frac{(1-\alpha')(2\alpha'+\gamma)-\sqrt{(1-\alpha')^2(2\alpha'+\gamma)^2-4\alpha'(1-\alpha')[(1-\alpha')(\alpha'+\gamma)-m]}}{2\alpha'(1-\alpha')}$$

(2) If $\gamma>m$, ① if $\gamma>2\sqrt{m}-1$, when firm 2 chooses modern level of copyright protection, i) when $\beta<\beta_3$, firm 1 will take strategy II, ii) when $\beta\ge\beta_3$, firm 1 will take a low level of copyright protection, and if firm 2 chooses a high or low level of copyright protection, firm 1 will take a low level of copyright protection. Where

$$\beta_3=\frac{(1-\alpha')(2\alpha'+\gamma)-\sqrt{(1-\alpha')^2(2\alpha'+\gamma)^2-4\alpha'(1-\alpha')[(1-\alpha')(\alpha'+\gamma)-m]}}{2\alpha'(1-\alpha')}$$

② if $\gamma<2\sqrt{m}-1$, whatever copyright protection firm 2 chooses, firm 1 will choose a low level of copyright protection.

In Duopoly, when one firm chooses no taking any copyright protection, and if the network effect is strong and the consumers who do not support piracy are less of one half, another firm will choose copyright protection. Or if the network effect is weak and the consumers who do not support piracy exceed one half, another firm will choose a low level of copyright protection. So we know that if one firm chooses copyright protection strategy first, then another also chooses protecting, only some time a low level of copyright protection. That is because weak copyright protection can refuse some price-sensitive

consumers, decreasing the price competition. In a word, the point of this game theory is the two firms all choose strategy II.

There are some difference between the situation of Oligopoly and Duopoly in a market. If there is only one firm (firm 1) in market, if $\beta > \frac{1+\gamma}{2}$, firm 1 will not choose copyright protection. However if there is another firm (firm 2) in the market, under the same condition, whatever copyright protection firm 2 chooses, firm 1 always chooses copyright protection. We assume that when there is only one firm in a market, this firm occupies the whole consumers who support the legal copy. If another firm joins in, it is inevitable to aggravate the market com- petition, and the share of consumers is becoming low. Form our model, the price of firm's product maybe increases. And this will refuse some price-sensitive consumers. So firms will take copyright protection to get more consumers to get more benefits.

## CONCLUSION

This paper examines how to choose copyright protection strategy under considering network effect and customers category. And we compare two strategy: no copyright protection and setting copyright protection. Then we divide the consumers into two parts: no-support-piracy consumers and support- piracy consumers. First we consider a mono- poly setting, our results suggest that stronger network effect is, the firm will prefer to choose setting copyright protection. And if the first segment of initial consumers is beyond one half and network effect is weak, the firm prefer to not set copyright protection. In Duopoly, we get that the point of game is two firm choose copyright protection.

This paper has done some work about the choice of copyright protection with net- work effect and customers category, and generates some conclusion. But the model of this paper is strict in assumption, because of the demand of simplifying research. And in real market, there are lots of other situation, such as when the firms have some products with different quality, the equilibrium of market will change. So future research can examine how different quality affect the choice of copyright protection of firms.

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# APPENDIX

## Proof of Proposition 1

We have

$$\pi_{11}^{*}-\pi_{12}^{*}=((1-\beta)^2+m)\alpha^2+(2\beta-1-\gamma)\bullet(1-\beta)\alpha,$$

Which we denote as $f(\alpha)$. This is a Quad- ratic function about $\alpha$, and is pointing up.

(1) When $2\beta-1-\gamma>0$, we know $f(\alpha)>0$, where $\alpha\in(0,1)$.

(2) When $2\beta-1-\gamma\le 0$, the symmetrical axis of $f(\alpha)$ is in the positive part. Now we discuss the value of function $f(1)$ is plus or not. ① If the value is plus, which can notes as $-\beta^2+(1+\gamma)\beta-\gamma+m>0$.

When $\beta=1$, then $f(1)=m>0$. And if $\beta=\frac{1+\gamma}{2}$, we know $f(1)>0$. So when $\gamma<m$, and $\beta\in(0,1)$, we can get $f(1)>0$. when $\gamma>m$, and $\beta_1<\beta<1$, we know $f(1)>0$. So we have known that when $\alpha\in(0,\alpha_1)$, where $\alpha_1\in(0,1)$, there will exist $f(\alpha)<0$. When $\alpha\in(\alpha_1,1)$, there be $f(\alpha)>0$. Where

$$\alpha_1=\frac{(1+\gamma-2\beta)(1-\beta)}{m+(1-\beta)^2}$$

② If the value is minus, which can notes as $-\beta^2+(1+\gamma)\beta-\gamma+m<0$.

So when $\gamma>m$ and $0<\beta<\beta_1$, we know $f(1)<0$. And in this condition when $\alpha\in(0,1)$, we have $f(\alpha)<0$. Where

$$\beta_1=\frac{1+\gamma-\sqrt{(1+\gamma)^2+4(m-\gamma)}}{2}$$

## Proof of Proposition 2

We have

$$\pi_{221}^{*}-\pi_{211}^{*}=-((1-\beta)^2+m)\alpha^2+(1+\gamma-\beta)(1-\beta)\alpha,$$

Where $\alpha$ is decision variable. Which we denote as $\varphi(\alpha)$. And $\varphi(\alpha)$ is a Quadratic function about $\alpha$, and is pointing downward. Symmetrical axis of $\varphi(\alpha)$ is in the positive part.

Now we discuss the value of function $\varphi(1)$ is plus or not. And

$$\varphi(1)=-((1-\beta)^2+m)+(1+\gamma-\beta)(1-\beta)=\gamma(1-\beta)-\mathrm{m} \quad (12)$$

(1) If the value is plus. Only when $\gamma>m$. So we get that when $\beta<1-\frac{m}{\gamma}$, always existing $\pi_{221}^{*}>\pi_{211}^{*}$;

(2) If the value is minus, we have $\beta>1-\frac{m}{\gamma}$. If $\gamma\le m$, then $\beta\in(0,1)$; if $\gamma>m$, then $\beta\in(1-\frac{m}{\gamma},1)$. Under these conditions, we get when $\alpha\in(0,\alpha_2)$, then $\pi_{221}^{*}>\pi_{211}^{*}$; when $\alpha\in(\alpha_2,1)$, then $\pi_{221}^{*}<\pi_{211}^{*}$.

Where $\alpha_2=\frac{(1+\gamma-\beta)(1-\beta)}{(1-\beta)^2+m}\in(0,1)$

## Proof of Proposition 3

We have

$$\pi_{221}'^{*}-\pi_{211}^{*}=-(m+(1-\beta)^2(1-\alpha')^2)\alpha''^2+(1+\gamma-\beta)(1-\beta)(1-\alpha')\alpha''$$

Where $\alpha''$ is decision variable. Which we denote as $g(\alpha'')$. And $g(\alpha'')$ is a Quadratic function about $\alpha$, and is pointing downward. The symmetrical axis of $g(\alpha'')$ is in the positive part. We know $g(0)=0$, and

$$\begin{aligned}g(1)&=-(m+(1-\beta)^2(1-\alpha')^2)+(1+\gamma-\beta)(1-\beta)(1-\alpha')\\&=(1-\alpha')\alpha'\beta^2-(2(1-\alpha')\alpha'+(1-\alpha')\gamma)\beta+(\gamma+\alpha')(1-\alpha')-m\end{aligned} \quad (13)$$

Which we denote as $h(\beta)$, and is pointing downward. The symmetrical axis of it is on the right side of $\beta=1$. And $h(1)=-m<0$ .$h(0)=(\gamma+\alpha')(1-\alpha')-m$.

$$=-\alpha'^2+(1-\gamma)\alpha'+\gamma-m \quad (14)$$

We denote $h(0)$ as $l(\alpha')$, and it is poi- nting downward. The symmetrical axis of $l(\alpha')$ is between 0 and 1. And we know

$$l(1)=-m<0\,,l(0)=\gamma-m$$

(1) when $\gamma>m$, we can get: ① there existing $\alpha_1'$ satisfy that when $\alpha'\in(0,\alpha_1')$, we have $h(0)>0$. If $\beta\in(0,\beta_2)$, then $g(1)>0$, then $g(\alpha'')>0$; if $\beta\in(\beta_2,1)$, then $g(1)>0$. Then there exist $\alpha_1''$ satisfy when $\alpha''\in(0,\alpha_1'')$, we have $g(\alpha'')>0$. ②when $\alpha'\in(\alpha_1',1)$, then $h(0)<0$, so we have $g(1)<0$. We also can get $\alpha_1''$ satisfying that when $\alpha''\in(0,\alpha_1'')$, then $g(\alpha'')>0$. Where

$$\alpha_1'=\frac{1-\gamma+\sqrt{(1-\gamma)^2+4(\gamma-\mathrm{m})}}{2}\text{、}$$

$$\alpha_1''=\frac{(1+\gamma-\beta)(1-\beta)(1-\alpha')}{m+(1-\beta)^2(1-\alpha')^2}\text{、}$$

$$\beta_2=\frac{(1-\alpha')(2\alpha'+\gamma)-\sqrt{(1-\alpha')^2(2\alpha'+\gamma)^2-4\alpha'(1-\alpha')[(1-\alpha')(\alpha'+\gamma)-m]}}{2\alpha'(1-\alpha')}$$

(2) when $\gamma<m$, we have: ① when $\gamma>2\sqrt{m}-1$ or $\gamma>0$, we know $l(\alpha')=0$ is solvable. Then when $\alpha'\in(\alpha_2',\alpha_3')$, we have $h(0)>0$. Where $\alpha_1'$, $\alpha_3'$ are two roots of $l(\alpha')$. And there exists $\beta_3$, which make when $\beta\in(0,\beta_3)$, there exists $g(1)>0$. then we have $g(\alpha'')>0$; when $\alpha'\in(0,\alpha_1')$ or $\alpha'\in(\alpha_1',1)$, we get $h(0)<0$. So if $\beta\in(0,1)$, then $g(1)<0$. So there exists $\alpha_2''$, which can satisfy that when $\alpha''\in(0,\alpha_2'')$, the equation $g(\alpha'')>0$ is setup. Where

$$\alpha_2'=\frac{1-\gamma-\sqrt{(1-\gamma)^2+4(\gamma-\mathrm{m})}}{2}\text{、}$$

$$\alpha_3' = \frac{1-\gamma+\sqrt{(1-\gamma)^2+4(\gamma-\mathrm{m})}}{2}$$、

$$\alpha_2'' = \frac{(1+\gamma-\beta)(1-\beta)(1-\alpha')}{m+(1-\beta)^2(1-\alpha')^2}$$、

$$\beta_3 = \frac{(1-\alpha')(2\alpha'+\gamma)-\sqrt{(1-\alpha')^2(2\alpha'+\gamma)^2-4\alpha'(1-\alpha')[(1-\alpha')(\alpha'+\gamma)-m]}}{2\alpha'(1-\alpha')}$$

② when $\gamma < 2\sqrt{m}-1$, $l(\alpha')$=0 is unsolvable. Then when $\alpha' \in (0,1)$, $h(0)$<0 sets up. And when $\beta \in (0,1)$, then we have $g(1)$<0. So there exists $\alpha_3''$, which makes when $\alpha'' \in (0,\alpha_3'')$, there will exist $g(\alpha'')$>0. Where

$$\alpha_3'' = \frac{(1+\gamma-\beta)(1-\beta)(1-\alpha')}{m+(1-\beta)^2(1-\alpha')^2}$$

And the premise of all of ② is $m \geq \frac{1}{4}$, so when $m < \frac{1}{4}$, there only exist the situation ①.